Too few modal integers
makes modular quotients multi-valued

Generating a cluster by multiplying its center times all the system's euler units is inefficient because it generates the whole cluster multiple times. That can become a serious issue for huge clusters with more than billions of members.

Notice in the Z_{12} example, for instance, that C_1 has twice as many members as C_3, so: $C_3=3\cdot\{1, 5, 7, 11\}$ but — more efficiently — $C_3=3\cdot\{a, b\}$ where a is either 1 or 5 and b is either 7 or 11: That only takes *half* as many multiplications as the inefficient method.

But...which half (or halves)? The answer to that is inseparable from the problem of modular division: What are the solutions q of a congruence like $d\cdot q \equiv n$? Hint: We would like to say $q \equiv n/d$, but there is usually more than one qualifying value for q! In particular, $3\cdot e \equiv 3 \pmod{12}$ has euler solutions $e \equiv 1$ or 5, and $3\cdot e \equiv 9$ has euler solutions $e \equiv 7$ or 11.

It gets worse if we drop the euler requirement; then $3\cdot q \equiv 3$ has solutions $q \equiv 1, 5,$ or 9 and $3\cdot q \equiv 9$ has solutions $q \equiv 3, 7,$ or 11.

With rare exceptions, all modal clusters except the euler cluster are smaller than the euler cluster, so each residue in each non-euler cluster is the product of the center with two or more distinct eulers — and residue arithmetic does not keep track: There is no way to determine which euler is responsible for any particular residue. Since division is an attempt to backtrack from quotient and denominator to the originating numerator, the absence of that information renders quotients somewhat ambiguous.

We address this division issue more sharply in due course.

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