Uniqueness Achieved

(i think) (**rethinking 3/13/2022**: maybe uniqueness was just a flash in the pan?)

There is no ambiguity in the center c nor integer i of the factorizations (c;e) or (i;u), nor in the base units e_{r_o} or u_{r_o} which are $e_{r_o} = r \;.\div.\; c_r$ or $u_{r_o} = r \;.\div.\; i_r$. They are fixed properties of r. But…

THOROUGH RE-DO 3/13/2022

There does exists ambiguity in the final choice of w in $c_r \cdot w \equiv c_r$ or $i_r \cdot w \equiv i_r$ where the number of alternatives equals the integer value of c_r or i_r. This translates into ambiguity in the unit factors of i_r, and it might translate to ambiguity in final quotients q:

Definition: $q \equiv_{(i;u)} \dfrac{n}{d}$ is defined by first (i;u)-factoring numerator and denominator as $n \equiv i_n \cdot u_n$ and $d \equiv i_d \cdot u_d$, then unit-dividing to get $q_u \equiv \dfrac{u_n}{u_d} \equiv u_n \cdot u_d{}^{-1}$ and euclidean-dividing i_n by i_d to get $q_i = i_n \;.\div.\; i_d$, then multiplying q_u by q_i to conclude with $q \equiv_{(i;u)} q_u \cdot q_i$.

Lemma: If integer i is a euclidean divisor of integer j, and w is any unit solution of $w \cdot i \equiv i$, then: $w \cdot i \equiv i$, $w \cdot j \equiv j$, $w^{-1} \cdot i \equiv i$, and $w^{-1} \cdot j \equiv j$ are all true.

Proof: Since $i \;.|.\; j$ we can let integer $k = j \;.\div.\; i$ so $j = i \cdot k$.

Then $w \cdot j \equiv w \cdot i \cdot k \equiv i \cdot k \equiv j$

so $w \cdot j \equiv j$.

Also $w^{-1} \cdot i \equiv w^{-1} \cdot (w \cdot i) \equiv \cdot (w^{-1} \cdot w) \cdot i \equiv \omega_w \cdot i$

so $w^{-1} \cdot i \equiv \omega_w \cdot (w \cdot i) \equiv (\omega_w \cdot w) \cdot i \equiv w \cdot i \equiv i$

so $w^{-1} \cdot i \equiv i$.

Proof that $w^{-1} \cdot j \equiv j$ proceeds similarly from . QED ▪

The reason the Lemma matters is that it explains why the factorization-style of modal division yields unique results: If denominator d divides numerator n then i_d divides i_n so the numerator n has the same aspects of d that make d transparent to the solutions w of $w \cdot d \equiv d$.